# **MULTI-RESOLUTION CORNER DETECTION**

Federico Pedersini, Elena Pozzoli, Augusto Sarti, Stefano Tubaro

Dip. di Elettronica e Informazione (DEI), Politecnico di Milano Piazza L. Da Vinci 32, 20133 Milano, Italy E-mail: pedersin/sarti/tubaro@elet.polimi.i

### ABSTRACT

In this paper we propose a novel technique for waveletbased multi-resolution corner detection. The method is based on the search of the zero crossings of the laplacian at different scales along the line that describes the trajectory of the maxima as the scale varies. The proposed techniques allows us to achieve sub-pixel accuracy and provides us with useful scale information on the detected features.

#### 1. INTRODUCTION

One crucial problem in the 3D reconstruction of scenes from uncalibrated views is the accurate detection and localization of point-like image features. When the features to be detected are markers that have been deliberately added to the imaged scene, a common choice is to perform spot detection through template matching. However, if the goal is to minimize the invasivity of the acquisition session, we need to search for natural image features such as corners or vertices. The literature is rich with techniques for corner extraction, which can be broadly classified into binary and grayscale. Binary algorithms are based on the analysis of the local curvature of luminance edges, while grayscale ones work by analyzing the output of some differential operators (gradient, curvature, hessian, etc.) applied to the luminance profile.

In order to achieve good quality in the 3D modeling of scenes from feature matching, feature localization needs to be as accurate as possible. More precisely, it would be desirable to maximize the (sub-pixel) localization accuracy, while minimizing the likelihood of extracting "false features" and keeping the probability of missing significant corners low.

When dealing with problems of stereometric reconstruction in the absence of temporal continuity (sets of snapshots), the feature matching problem becomes rather difficult to assess, especially if the acquisition conditions of each image are drastically different (local high-resolution views vs. global low-resolution views). In this case, scale/resolution information on the detected features could constitute a source of precious information, which could speed up the matching process and make it more reliable and robust. Knowing the scale level that a feature corresponds to, can also be useful in order to understand more about the geometry of the feature itself. For example, the scale depth could tell us something about how "local" the considered feature is (in the case of edges, how pronounced and isolated the transition is, while in the case of corners, how isolated and how long the crossing edges are). Finally, a multiresolution approach reduces the likelyhood of false corner extraction as, at each resolution level, the algorithm focuses on the detection of just the corners of that specific scale.

The importance of multiscale detection of features is widely recognized in the image analysis and the computer vision communities. In fact, a number of solutions have already been proposed for both edges and corners. Particularly interesting is a wavelet-based technique for multiscale edge detection [3] that generalizes a classical approach due to Canny [4]. In a quite similarl fashion, our approach to corner detection is a wavelet-based multiresolution generalization of the grayscale approach proposed by Deriche and Giraudon [1], which is based on the analysis of the multiscale Laplacian of the luminance profile. Being a grayscale method, it allows us to estimate the corner's location with sub-pixel accuracy even when the corners are very "localized" on the image.

#### 2. LAPLACIAN-BASED CORNER DETECTION

It is well known that a corner modeled as a crossing between smoothed step edges is a zero-crossing point for the Laplacian, irrespective of its orientation [2]. In addition, there is another differential operator the determinant of the Hessian matrix operator of the luminance profile f(x, y), is known to exhibit a relative maximum in the proximity of the corner. As we progressively filter the image, this maximum shifts along a line that passes through the corner. These two facts can be jointly used in order to determine a corner with super-resolution accuracy [1]. In order to do so, we can search for a zero-crossing of the DET along the line of the DET's maxima (see Fig. 1). It is important to notice, however, that also the Laplacian of the luminance profile exhibits a relative maximum in the proximity of the corner, which shifts along a line that passes through the corner as we progressively filter the image. This fact suggests us that corner extraction can be achieved using just the Laplacian operator (see Fig. 2).



**Fig. 1.** Illustration of the Laplacian-based corner detection mechanism: the curves represent the zero crossings of the Laplacian while the line is the locus of the maxima of either the DET or the Laplacian. The corner is a zero crossing of the Laplacian along the line of maxima.

#### 3. MULTISCALE LAPLACIAN

In order to make the corner extraction multiscale, we derive here a wavelet decomposition that extracts the multiscale Laplacian. With reference to Fig. 3, the herringbone structure that implements the fast wavelet transform (FWT) computes the second-order derivatives  $f_{xx}^j$  and  $f_{yy}^j$  at the various scales j = 1, 2, ..., which can be added together to provide the Laplacian  $\nabla^2 f^j = f_{xx}^j + f_{yy}^j$ . Notice that only the side transforms  $f_{xx}$  and  $f_{yy}$  need to be computed, with a significant computational saving.

Let us first consider a 1D basic wavelet  $\psi(x)$  defined as  $\psi(x) = \frac{\partial^2 \theta(x)}{\partial x^2}$ , where  $\theta(x)$  is a (low-pass) smoothing function. With this choice, the wavelet transform of f(x) is

$$W_{2^{j}}f(x) = f * \psi(x) = f * \frac{\partial^{2}\theta(x)}{\partial x^{2}} = \frac{\partial^{2}}{\partial x^{2}}(f * \theta(x)) \quad ,$$
(1)



**Fig. 2**. Surface profile of the DET (left) and of the Laplacian (right) in the neighborhood of a corner: both exhibit a maximum.



**Fig. 3.** Wavelet decomposition for the multiscale computation of the Laplacian. At each scale the side subbands represent the second-order derivatives of the filtered image.

which is the second derivative of a filtered version of f(x). Similarly, in the 2D case we have

$$W_{2j}^{1}f + W_{2j}^{2}f = f * \psi_{2j}^{1} + f * \psi_{2j}^{2}$$
  
=  $2^{j} \left( \frac{\partial^{2}}{\partial x^{2}} (f * \theta_{2j}) + \frac{\partial^{2}}{\partial y^{2}} (f * \theta_{2j}) \right)$   
=  $2^{j} \nabla^{2} (f * \theta_{2j})$ , (2)

which allows us to compute the components of the Laplacian as a simple sum of two of the three wavelet components of the Transform.

Similarly to what seen in [3], one wavelet that satisfies the above requisites is the B-spline. As we know, in the 1D cubic spline is defined as

$$\theta(x) = \begin{cases} 8|x|^3 - 8|x|^2 + \frac{4}{3} & , |x| \le 0, 5\\ -\frac{8}{3}|x|^3 + 8|x|^2 - 8|x| + \frac{8}{3} & , 0, 5 \le |x| \le 1\\ 0 & , |x| > 1 \end{cases}$$
(3)

therefore the corresponding wavelet results as

$$\psi(x) = \frac{d^2\theta(x)}{dx^2} = \begin{cases} 48 |x| - 16 & , \quad |x| \le 0, 5\\ -16 |x| + 16 & , \quad 0, 5 \le |x| \le 1 \\ 0 & , \quad |x| > 1 \end{cases}$$
(4)

which, as expected, turns out to be a linear spline. The corresponding scaling function  $\phi(x)$  is thus

$$\phi(x) = \begin{cases} -x+1 & , & 0 \le x \le 1 \\ x+1 & , & -1 \le x \le 0 \end{cases}$$
(5)

As far as the (LP) scaling filter  $h_n$  and the (HP) wavelet filter  $g_n$  are concerned, they must satisfy the conditions

$$\begin{split} \phi(x) &= \sum_{n} h_n \phi(2x-n) \\ \psi(x) &= \sum_{n} g_n \phi(2x-n) , \end{split}$$

therefore we have  $h_{-1} = \frac{1}{4}$ ,  $h_0 = \frac{1}{2}$ ,  $h_1 = \frac{1}{4}$ ; and  $g_{-1} = 4$ ,  $g_0 = -8$ ,  $g_1 = 4$ ; which correspond to the transfer functions  $H(\omega) = (1 + \cos(\omega))/2$  and  $G(\omega) = 8(-1 + \cos(\omega))$ . In the 2D case, in order to minimize the computational complexity, we choose a separable scaling function of the form  $\phi(x, y) = \phi(x)\phi(y)$ . As for the *wavelets*, we have

$$egin{array}{rcl} \psi^1(x,y)&=&\partial^2 heta^1(x,y)/\partial x^2=\psi(x)2\phi(2y)\ \psi^2(x,y)&=&\partial^2 heta^2(x,y)/\partial y^2=2\phi(2x)\psi(y) \ . \end{array}$$

where  $\theta^1(x, y) = \theta(x)2\phi(2y)$  and  $\theta^2(x, y) = 2\phi(2x)\theta(y)$ have approximately the same shape. The corresponding 2D scaling function is

### 4. MULTISCALE CORNER DETECTION

The global structure of our multiscale corner extraction algorithm can be summarized as in Fig. 4. The first step is the computation of the multiscale Laplacian through Fast Wavelet Transform (FWT). The side transforms of each scale level are then added together to generate the multiscale Laplacian. The maxima of the Laplacian are then detected (with subpixel accuracy) while tracked along scale changes in order to compute the lines of maxima. Each line of maxima is determined through a process of iterative minimization of the weighed square distance between line and maxima, the weights being indices of reliability based on their local symmetry. Once the line of maxima is available for a given corner, the zero crossing of the Laplacian is searched for along this line with subpixel accuracy (zero-crossing of the cubic interpolation of the maxima along the line).

## 4.1. Corner Validation

Corner validation is performed through a check on the local variance of the luminance profile. Residual false corners could still be detected along straight edges at high resolution. Such corners can be easily removed the an analysis of the 1D luminance profile obtained by scanning the 2D profile along a circle centered on the corners under examination (see Fig. 5). Notice that this last step tends to reduce the likelihood of false corner detection, but it also tends to reduce the number of good corners that are, in fact, detected. In some applications, such as uncalibrated 3D reconstruction from multiple views, false features can be better removed through some robust multi-view analysis approach. In this case, it is preferable to maximize the detection rate and skip the outlier's removal step of Fig. 4.

## 4.2. Accuracy

The proposed algorithm returns the location of corners with a localization accuracy of the order of the fraction of pixel (usually 0.2-0.4 pixel with natural images). The inter-scale delocalization is normally below 0.1 pixel between successive scales.

## 5. CONCLUSIONS

In this paper we proposed a novel method for the multiscale detection and localization of corners based on the analysis of the multiscale laplacian's profile. The method has proven effective for an accurate localization of corners, and has shown to provide valuable information on the "depth of scale" with a low probability of missing corners or detecting false vertices. Further research is being done in order to improve the isotropy of the multiscale Laplacian extraction filter and, therefore, its accuracy.

### 6. REFERENCES

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Fig. 4. Global structure of the algorithm.



**Fig. 5.** Outlier's elimination: false corners are removed through the analysis of the luminance profile along a circle centered on the corner itself.

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Fig. 6. From top to bottom: corners extracted as the scale increases.