

FROM DIRECTION OF ARRIVAL ESTIMATES TO LOCALIZATION OF PLANAR REFLECTORS IN A TWO DIMENSIONAL GEOMETRY

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ABSTRACT

In this paper we propose a novel technique to localize planar obstacles through the measurement of the Direction of Arrival by a microphone array. The measurement of the Direction of Arrival of the reflected path is turned into a quadratic constraint where the unknowns are the line parameters of the reflector. A cost function that combines multiple constraints is then derived. A parametric description of the obstacle is found by minimization of the cost function. Some simulations and experimental results show the feasibility of the proposed method.

Index Terms— Direction of Arrival, Microphone Array, Reflector Localization

1. INTRODUCTION

The knowledge of the acoustic properties of the environment is crucial for many space-time processing applications. As an example, rendering of soundfields through loudspeaker arrays could greatly benefit from the knowledge of the location of obstacles in space. In [1] the authors adopt a parametrization of the acoustic transfer function of the environment in which the loudspeaker array operates to compensate the effect of reverberation. A good knowledge of the geometry of the environment along with its acoustic properties are therefore needed to compensate the effect of reverberations. In this paper we propose a technique for the localization of a planar obstacle in space that is based on the measurement of the Direction of Arrival of the reflective path with a circular microphone array.

Techniques for the reconstruction of the geometry of the environment are common in computer vision. However, due to the different wavelengths of sound and optical waves, computer vision techniques return a geometry of the environment which contains too much detail for acoustic purposes and, moreover, reflectance properties of materials can strongly differ between the acoustic and optical domains. For this reason the use of acoustic stimuli for the reconstruction of the geometry of the environment is highly desirable.

In [2] the authors use a constrained room model and a ℓ_1 least-squares regularization to perform the estimation of the room geometry from the acoustic impulse responses. In [3] the authors present a technique for the estimation of the reflective surfaces from continuous signals in theaters and large auditory rooms, which is based on inverse mapping of the acoustic multi-path propagation problem. In

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[4] the authors present a solution for the localization of 2D reflectors based on robust beamforming. In [5] the reconstruction of the geometry of the environment is performed through the analysis of the impulse responses from different source-microphone pairs. The Times Of Arrival (TOAs) of the reflected path are then converted into quadratic constraints, in which the line on which the reflector lies is bound to be tangential to an ellipse with foci in microphone and source and with major diameter proportional to the Time Of Arrival. Through the combination of multiple constraints a cost function is derived.

All the methods presented above are based, however, on the measurement of the Time Of Arrival of the reflected path, which implicitly requires the synchronization between source and microphones, which is not viable in many applications. In this paper we localize planar obstacles through measurements of Direction Of Arrival of the reflected path for different positions of the acoustic source. More specifically, given the location of the source and the DOA of the reflected path, the line that parameterizes the position of the obstacle is constrained to be tangential to a parabola having the focus in the source and directrix the measured DOA. A parabola is easily described by its matrix quadratic form, as for ellipses. The same approach used in [5] for the definition of a cost function as combination of multiple constraints can be used. However, with respect to [5], we propose here an improvement in the minimization process of the cost function. In a future step, we envision to remove the assumption of the knowledge of the source location by estimating it using a suitable approach.

The rest of the paper is structured as follows: Section 2 summarizes the procedure used throughout the paper to measure the DOA. Section 3 derives the constraint related to DOA. Section 4 presents the combination of multiple constraints into a cost function and discusses through some simulations the robustness of the methodology to errors in the measurement of the DOA. Finally, Section 5 shows some simulations and experimental results to validate the proposed methodology.

2. PROBLEM FORMULATION AND MEASUREMENT OF DOA

In this paper we assume that the geometry of the environment is two-dimensional, where a planar obstacle is represented by a line. Consider the presence of a circular microphone array centered in the origin of the reference frame. A generalization to a three-dimensional geometry is still possible without main modifications. A source is located in the known position \mathbf{x}_s . The reflector is parameterized by the parameters of the line on which it lies, i.e. a point $\mathbf{x} = [x_1, x_2]^T$

lies on the reflector line $\mathbf{l} = [l_1, l_2, l_3]^T$ if and only if

$$l_1 x_1 + l_2 x_2 + l_3 = 0.$$

Our goal is to estimate the vector \mathbf{l} from the knowledge of the positions $\mathbf{x}_n, n = 1, \dots, N$ of the acoustic source and from the measurement of the DOAs $\theta_n, n = 1 \dots, N$ of the reflected signal for each source position, where $N \geq 3$. Figure 1 summarizes the geometry of the problem.

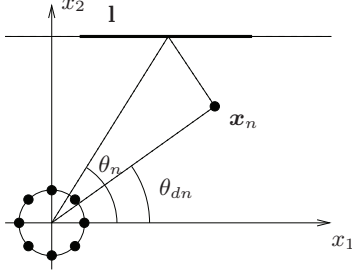


Fig. 1. Problem formulation: an acoustic source is located in \mathbf{x}_n , the planar reflector is parameterized by the line parameters \mathbf{l} . The DOAs related to the direct and reflected signals are θ_{dn} and θ_n , respectively.

2.1. Measurement of the DOA

We now summarize the algorithm used throughout the paper to measure the Direction of Arrival. The i th microphone in the array acquires the signal $s_i(t)$, where t is the discrete time-index. The symbol $s_i(t, \omega_k)$ denotes the filterbank analysis of $s_i(t)$ in the sub-band centered in $\omega_k, k = 1, \dots, K$. The signals are organized in the column vector $\mathbf{s}(\omega_k, t)$, which stacks the signals for all the microphones in the array. In order to estimate the Direction Of Arrival we make use of the wideband Capon algorithm proposed in [6]. The frequency-dependent autocorrelation matrix is computed as

$$\mathbf{R}_k = \frac{1}{K} \sum_{t=1}^T \mathbf{s}(\omega_k, t) \mathbf{s}(\omega_k, t)^H, k = 1 \dots, K. \quad (1)$$

For each sub-band, the pseudospectrum $f(\theta, k)$ is computed as

$$f(\theta, k) = \frac{1}{\mathbf{a}(\theta, k)^H \mathbf{R}_k \mathbf{a}(\theta, k)}, \quad (2)$$

where $\mathbf{a}(\theta, k)$ is the propagation vector for the microphone array for the angle θ and the sub-band k . The overall pseudo-spectrum is computed as the geometric mean of the pseudo-spectra of the different sub-bands [6]

$$F(\theta) = \left(\prod_{k=1}^K f(\theta, k) \right)^{\frac{1}{K}}. \quad (3)$$

Finally, the DOAs are selected as the two angles of the most relevant local maxima of $F(\theta)$. The DOA related to the direct signal is then discarded as we know the position of the source and we are able to remove it.

3. CONSTRAINT FROM DOA

In this Section we derive the quadratic constraint related to the DOA measurement of the reflected path. The DOA related to

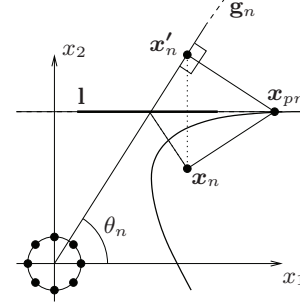


Fig. 2. An acoustic source at \mathbf{x}_n is mirrored over the reflector to its image position \mathbf{x}'_n . A sensor array centered in the origin measures the DOA θ_n related to the reflected path.

the reflected path is θ_n , and it unequivocally defines the line $\mathbf{g}_n = [\sin(\theta_n), -\cos(\theta_n), 0]^T$ passing through the centre of the microphone array and with direction θ_n . We can think of the reflective path as generated by the image source \mathbf{x}'_n , obtained by mirroring the source $\mathbf{x}_n = [x_{1n}, x_{2n}]^T$ against the reflector. We notice that if we make an hypothesis on the position of the image source \mathbf{x}'_n , the reflector line \mathbf{l} is constrained to be the axis of the segment $\mathbf{x}_n \mathbf{x}'_n$ in order to honor the Snell's law. Let \mathbf{x}_{pn} be the intersection between the line perpendicular to \mathbf{g}_n through \mathbf{x}'_n and the reflector line \mathbf{l} . By construction the triangle $\mathbf{x}_n \mathbf{x}'_n \mathbf{x}_{pn}$ is isosceles and \mathbf{l} is the bisector of the angle $\mathbf{x}_n \mathbf{x}_{pn} \mathbf{x}'_n$. We recall that the focal property of a parabola with focus \mathbf{x}_n and directrix \mathbf{g}_n states that the line tangent to a parabola at \mathbf{x}_{pn} is also the bisector of the angle formed by the line joining \mathbf{x}_n and \mathbf{x}_{pn} and the perpendicular to \mathbf{g}_n through \mathbf{x}_{pn} , as shown in Figure 2. The parabola is also described as the locus of points equidistant from \mathbf{g}_n and from \mathbf{x}_n , which is equivalent to write

$$|x_1 \sin \theta_n - x_2 \cos \theta_n| = \sqrt{(x_{1n} - x_1)^2 + (x_{2n} - x_2)^2}, \quad (4)$$

where the left side is the Euclidean distance between the point $\mathbf{x} = [x_1, x_2]^T$ and the directrix; the right side corresponds to the Euclidean distance between \mathbf{x} and the focus. Squaring both the sides and expanding the terms, after some calculation we obtain the following equivalent matrix formulation of eq.(4)

$$\mathbf{x}^T \mathbf{C}_n \mathbf{x} = 0 \quad (5)$$

where $\mathbf{x} = [\mathbf{x}, 1]^T$.

$$\mathbf{C}_n = \begin{bmatrix} \sin^2 \theta_n & -\sin \theta_n \cos \theta_n & -x_{1n} \\ -\sin \theta_n \cos \theta_n & \cos^2 \theta_n & -x_{2n} \\ -x_{1n} & -x_{2n} & x_{1n}^2 + x_{2n}^2 \end{bmatrix}. \quad (6)$$

We notice that the matrix \mathbf{C}_n has one negative and two positive eigenvalues. In order to formulate the constraint given by eq.(5) in terms of the reflector line \mathbf{l} , we consider the dual representation of the conic [5] given by

$$\mathbf{l}^T \mathbf{C}_n^* \mathbf{l} = 0, \quad (7)$$

where \mathbf{C}_n^* is the adjoint of the matrix \mathbf{C}_n . Eq.(7) states that the parabola with focus in \mathbf{x}_n and directrix \mathbf{g}_n is the envelope of all the candidate reflector lines.

4. REFLECTOR LOCALIZATION FROM CONSTRAINTS

In this Section we consider the localization of obstacles by combining the constraints in (7) for multiple positions of the source. Following the same approach used in [5], we define the cost function

$J(\mathbf{l})$ as the square sum of the residuals of the individual constraints

$$J(\mathbf{l}) = \sum_{n=1}^N \|\mathbf{l}^T \mathbf{C}_n^* \mathbf{l}\|^2. \quad (8)$$

The searched solution is the global minimum of (8),

$$\hat{\mathbf{l}} = \arg \min_{\mathbf{l}} J(\mathbf{l}). \quad (9)$$

The constraint in (7) is homogeneous, therefore the cost function $J(\mathbf{l})$ admits the trivial solution $\mathbf{l} = [0, 0, 0]^T$, which is not a valid line parameter. In order to prevent the minimum search to incur into this solution, we put a unitary norm constraint on the solution, therefore eq. (9) becomes

$$\hat{\mathbf{l}} = \arg \min_{\mathbf{l}} J(\mathbf{l}) \text{ subject to } \mathbf{l}^T \mathbf{l} = 1. \quad (10)$$

We notice that the constraint in (10) corresponds to finding the minimum of $J(\mathbf{l})$ on the surface of a unitary sphere in the line parameter space (l_1, l_2, l_3) , i.e.

$$\begin{cases} l_1 = \cos(\alpha) \\ l_2 = \sin(\alpha) \cos(\beta) \\ l_3 = \sin(\alpha) \sin(\beta) \end{cases}, \quad (11)$$

where α and β are the angles in spherical coordinates. With this substitution at hand, the searched minimum becomes

$$(\hat{\alpha}, \hat{\beta}) = \arg \min_{(\alpha, \beta)} \tilde{J}(\alpha, \beta), \quad (12)$$

where $\tilde{J}(\alpha, \beta)$ is the expression of the cost function in the previously defined angular coordinates. We perform the minimization in (12) using the Nonlinear Least Squares algorithm. In the next Section we analyze the impact of errors in the DOA measurement on the reflector localization.

5. SIMULATIONS AND EXPERIMENTS

This section is divided in three parts. First we introduce the metric used to evaluate the reflector line estimation. We then analyze by means of simulations the robustness of the reflector location against DOA measurement errors. Finally, we show the results of some experiments that show the feasibility of the proposed approach.

5.1. Evaluation metric

The metrics used throughout the paper to evaluate the quality of the line parameter estimation are

- the difference Δd of the distances of the real and estimated reflectors from the origin of the reference frame;
- the angular error $\Delta\phi$ between the real and the estimated reflectors.

5.2. Simulations

We simulate the measurement of noisy DOAs. Ten sources are equally located on circles of radius ρ between 0.5 and 1.5 m and centered in the center of the circular microphone array. The error on the DOA is simulated by adding a gaussian noise to the correct DOA. The standard deviation of the noise is variable in the range between 0.1 and 5 degrees. For each position of the source 10000 measurements have been simulated.

In the first simulation the line parameter vector of the reflector is $\mathbf{l} = [0, 1, -1.6]^T$. The results are shown in Figure 3. We notice that the angular error is almost independent from the distance of the source, while the distance error is higher for smaller distances of the source.

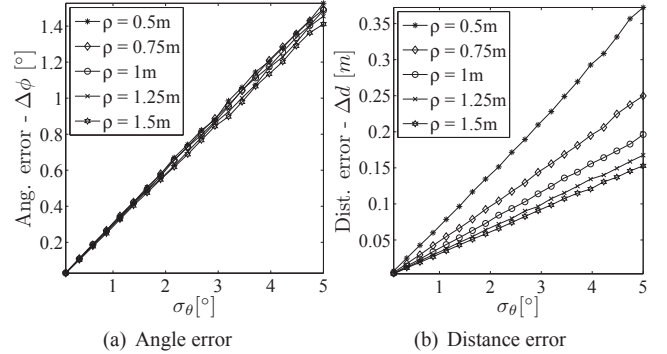


Fig. 3. Angular and distance errors for the reflector with line parameter $\mathbf{l} = [0, 1, -1.6]^T$ for different distances of the source from the microphone array.

In the second simulation the line parameter vector of the reflector is $\mathbf{l} = [0, 1, -2]^T$, i.e. the reflector is more distant from the center of the microphone array. The results are shown in Figure 4. As above, the angular error is almost independent from the distance of the source, while the distance error is higher for smaller distances. By comparing the two experiments, we notice that the error increases

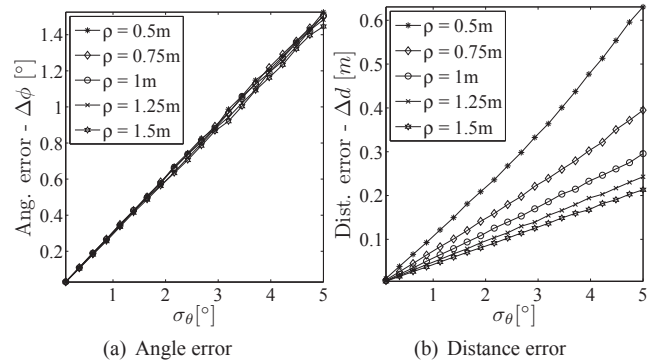


Fig. 4. Angular and distance errors for the reflector with line parameter $\mathbf{l} = [0, 1, -2]^T$ for different distances of the source from the microphone array.

with the distance of the reflector from the center of the microphone array.

5.3. Experiments

In the experiments we test the reflector localization accuracy as a function of the number of DOA measurements used in the estimation. The microphone array and the source lie on a horizontal plane, while the reflector is vertical. We adopt this configuration to reproduce a two-dimensional geometry. The microphone array is composed by 10 sensors disposed on a rigid cylindrical baffle with radius of 0.04 m, as in Figure 5. For further reference on the microphone array, see [7]. The signal acquired by the sensors is sampled



Fig. 5. The circular microphone array used in the experiments [7].

at $F_s = 44100$ Hz. In all the experiments we consider the presence of a single reflector in the environment. For this purpose, we place a planar reflective surface in a low-reverberation chamber. Figure 6 shows 140 potential positions of the source, disposed on a grid. However, due to the reciprocal configuration of the array and the reflector, not all the positions on the grid generate a reflective path. For this reason, we place the loudspeaker at 88 positions out of 140. The

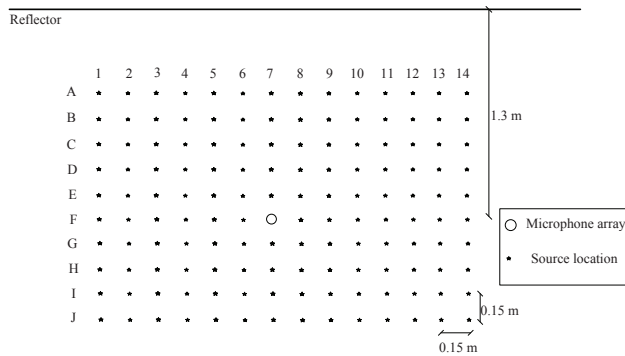


Fig. 6. Floor view of the experimental setup: the grid marks all the potential positions of the loudspeaker. The microphone array is at the center of the grid. The reflector is distant 1.3 m from the center of the microphone array.

loudspeaker emits a white noise in the band [1 kHz ~ 10 kHz]. We test the accuracy of the reflector localization (in terms of distance and angle error) as a function of the number of constraints N used in the minimization, N being between 3 and 24. For each N we show the average distance and angle error for all the possible combinations of DOA measurements. Figure 7 shows the average angular and distance error. We notice that even with three constraints, i.e. three source positions, the reflector is well localized. In order to show the stability of the estimation, we show also the standard deviation of the angle and distance errors as a function of the number of DOA measurements. We notice that as the number of constraints increases the standard deviation decreases, as expected. Nonetheless, even when $N \geq 4$ measurements are used, the solution is almost independent from the specific set of measures used.

6. CONCLUSIONS AND FUTURE WORKS

In this paper we proposed a methodology for the localization of planar obstacles, which relies on the measurements of DOAs. This approach does not require the synchronization between the acoustic source and the microphone array. Theoretical and experimental results prove the feasibility of the method, also in presence of relevant

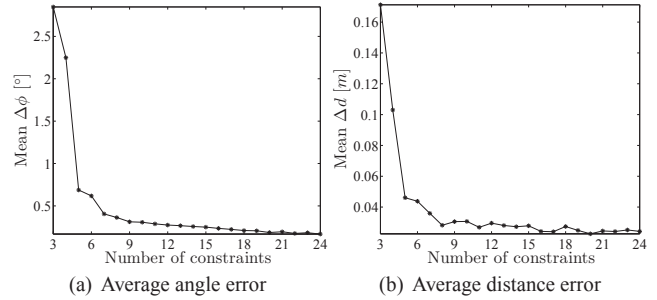


Fig. 7. Average angular and distance errors vs number of DOAs.

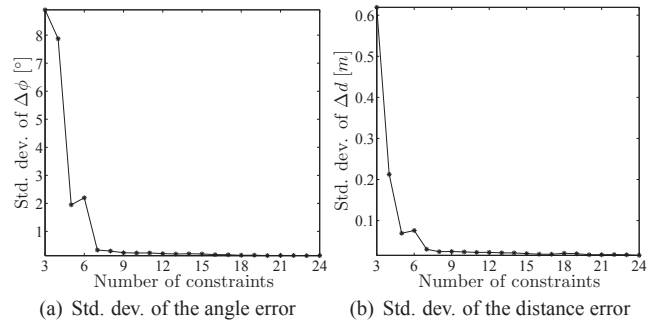


Fig. 8. Std. dev. of angular and distance errors vs number of DOAs.

error in the DOA measurement. We are currently working on the extension of the method to the localization of multiple reflectors and on the combination of multiple types of constraints into the same cost function.

7. REFERENCES

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